PMHT Path Planning in a non-Homogeneous Environment

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***Abstract*—This paper considers the problem of automatically coordinating multiple platforms to explore an unknown environment. A newly developed method for this problem consists of deterministically placing *locales* within the region and then using the Probabilistic Multi-Hypothesis Tracker (PMHT) to associate these locales with the platforms at specific times. The method uses an integer number of locales at discrete locations to guide the platforms. This paper extends the method to instead guide the platforms using a continuous valued non-uniform intensity map over a continuous spatial domain. The existing method is shown to be a discretised approximation to the continuous intensity map.**

I. INTRODUCTION

Multiple platform path planning is a problem that arises in many applications including search and rescue, coordinated surveillance, multiple platform simultaneous localisation and mapping (SLAM) and resource dissemination (the travelling salesman). Independent of the application, the goal is to schedule multiple mobile resources with dynamic constraints to cover an area in an efficient manner.

There are many strategies to coordinate a single moving platform to intelligently explore an environment. Most of these strategies take different factors into account before choosing an action, such as the localisation error of the platform's sensors or the information gain potential through the measurements that these sensors may collect. Generally these approaches enumerate a collection of hypotheses based on feasible motion of the platform and use a cost function as the decision criterion. For example, one strategy is to move the platform to build the map information as quickly as possible with localisation being less important, such as [1]. In contrast, under Active Localisation [2], the platform self localisation is most important and the platform's movement is chosen based on what will be likely to reduce the localisation uncertainty. Other methods seek to optimise the view points of the platform to maximise the expected information gain in building the map and to minimise the uncertainty of the platform within the grid cells of the map [3]. These strategies may be greedy one step ahead or N step ahead and are generally for single platforms.

It has been widely acknowledged within the autonomous vehicle research community that the use of multiple cooperating vehicles for exploration tasks has many advantages over a single vehicle architecture. Multiple vehicles have the potential to explore and map an environment more quickly than a single vehicle, are more robust to failures, and provide a broader field of view in dynamic environments. To achieve cooperative exploration, the key problem is to choose appropriate actions for the platforms so that they simultaneously explore different regions of the environment optimally. Similar to the single platform strategies, there are extensions to the multiple platform case, such as [4] to build a map as quickly as possible and [5] using information gain to coordinate the multiple platforms.

A new approach for multiple platform path planning was introduced in [6]. This method used the Probabilistic Multi-Hypothesis Tracker (PMHT) to design trajectories for the platforms based on known initial platform conditions (position, speed, heading). A set of *locales* was created to assist the planning, which were discrete locations within the area of interest. The assumption was that the original planning problem could be solved by constructing platform trajectories such that all of these locales were visited at least once.

The novelty in the PMHT path planning approach is to recognise that the path planning problem is mathematically equivalent to a tracking problem where the locales are analogous to measurements and the platforms are analogous to targets. This means that the problem can be solved using a multi-target tracking algorithm, where the data association capability of the tracker is used to decide which platform should visit each landmark and at which time. The tracker ensures that each of the locales is visited by at least one of the platforms, while constraining the motion of the platforms to a realistic dynamic model. The difficulty is that the locales have no intrinsic temporal relationship. Whereas in tracking the usual assumption is that the input information is noisy position estimates collected at known times, here there are no times associated with each locale; there is not even a preferred order in which to visit them. In order to overcome this problem, hypothesised time-stamps are treated as hidden variables and the PMHT is used to associate the locales to platforms and times simultaneously.

The PMHT, developed by Streit and Luginbuhl [7], is a data association algorithm derived from the application of the Expectation Maximisation (EM) algorithm [8] to target tracking. The PMHT uses EM to model the assignment of

978-1-4577-0674-5/11/$26.00

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
MAR 2012	**N/A**	**-**

4. TITLE AND SUBTITLE
PMHT Path Planning in a non-Homogeneous Environment

5a. CONTRACT NUMBER
5b. GRANT NUMBER
5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

5d. PROJECT NUMBER
5e. TASK NUMBER
5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Defence Science and Technology Organisation, AUSTRALIA

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR’S ACRONYM(S)

11. SPONSOR/MONITOR’S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
See also ADA559142. Proceedings of the International Conference on Intelligent Sensors, Sensor Networks and Information Processing (7th). Held in Adelaide, Australia, on 6-9 December 2011. Government or Federal Purpose Rights License., The original document contains color images.

14. ABSTRACT
AbstractThis paper considers the problem of automatically coordinating multiple platforms to explore an unknown environment. A newly developed method for this problem consists of deterministically placing locales within the region and then using the Probabilistic Multi-Hypothesis Tracker (PMHT) to associate these locales with the platforms at specific times. The method uses an integer number of locales at discrete locations to guide the platforms. This paper extends the method to instead guide the platforms using a continuous valued non-uniform intensity map over a continuous spatial domain. The existing method is shown to be a discretised approximation to the continuous intensity map.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			
unclassified	**unclassified**	**unclassified**	**SAR**	**6**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

measurements to targets as hidden variables and estimates target states by taking the expectation over the assignments. The advantage of the PMHT over alternative data association techniques is that it has linear complexity in the number of targets, the number of measurements per frame, and in the number of frames. In contrast, other data association methods incur a complexity that grows with the number of permutations of measurements and targets within a frame and then exponentially with time.

In the context of the planning problem, PMHT allows for batch data association over hundreds of time steps with multiple platforms in a field of hundreds of locales. It is also built on an EM framework which is amenable to the incorporation of the missing temporal information.

The initial work of [6] derived the PMHT path planning algorithm using the above notion of a set of discrete locales. Qualitative examples of the resulting paths were presented in [6] and the method was quantitatively compared with a genetic-algorithm alternative in [9]. This initial work generally used a uniform placement of locales, either deterministically placed in a grid, or randomly placed by sampling a uniform spatial distribution. In this paper, we generalise this driving model in two ways. Firstly we demonstrate how to incorporate a non-homogeneous (i.e. spatially non-uniform) locale set, and secondly this non-homogeneous discrete set of points is generalised to an arbitrary intensity function over a continuous-valued spatial area.

The resulting improvement to the PMHT path planner no longer uses the notion of artificially generated locales to guide the platform trajectories. Instead it is driven by an intensity function, which (for example) could be a probability-hypothesis-density (PHD) [10] representation of the number and spatial distribution of targets within a surveillance volume.

The remainder of this paper is arranged as follows: section II reviews the existing PMHT method for path planning using a uniformly weighted collection of artificial locales. Section III extends this to a non-uniformly weighted set of discrete locales by introducing the concept of locale *priority*. Section IV generalises this further by transforming the discrete locale grid into a continuous density. Section V provides a simulation example, and VI provides a summary.

II. PMHT FOR MULTIPLE PATH PLANNING

The PMHT-pp algorithm is an extension of the standard PMHT to address the problem of path planning for multiple platforms in a field of discrete locales [6]. The PMHT-pp derivation in [6] is very similar to that of the standard PMHT, presented in detail in [7] and [11]. The development of PMHT-pp is sketched here with more detail provided in areas that become important for the following sections. For a complete derivation, see the above references.

Let there be N locales, $z_1 \dots z_N$, with $Z = \{z_n\}_{n=1}^N$ denoting the whole locale set. Assume that there are M platforms and that they are to be guided amongst these locales over a time-horizon of T uniformly sampled time steps.

Let the state of platform m at time t be denoted x_t^m, the set of all states for platform m be denoted $X^m = \{x_t^m\}_{t=1}^T$, and the set of all states be denoted $X = \{X^m\}_{m=1}^M$. For the purpose of this paper, the state consists of position, velocity and acceleration in the plane.

It is assumed that the prior distribution of the state of each platform is known and is given by $\psi_0^m(x_0^m)$ for platform m. The platform dynamics are also assumed to be known and can be described by the evolution probability density function (pdf) $\psi_t^m(x_t^m|x_{t-1}^m)$.

To codify the decision of which platform should visit each locale and at which time slot, we introduce two hidden assignment variables. Let the platform assigned to locale n be $k_n \in \{1 \dots M\}$, with $K = \{k_n\}_{n=1}^N$ denote the set of all platform-to-locale assignments. Let the time at which locale n is visited by a platform be $\tau_n \in \{1 \dots T\}$ and $\tau = \{\tau_n\}_{n=1}^N$ denote the set of all time-to-locale assignments. Both of these are, of course, unknown. Treat both k_n and τ_n as random variables with priors π_{nm}^k and π_{nt}^τ respectively. These priors are also unknown. Π^k and Π^τ denote the collection of platform and time assignment priors respectively.

The path planning problem is solved by optimising a cost function that is mathematically equivalent to the log-likelihood of a tracking problem where the locales are instead random measurements and the platforms are targets. The cost of assigning locale n to platform m at time t is a function of the positions of the two at time t and is analogous to the measurement pdf in the tracking problem. Assume a linear Gaussian measurement pdf

$$\zeta(z_n|x_t^m) \sim N(z_n; Hx_t^m, R) \tag{1}$$

where $m = k_n$, $t = \tau_n$, H is the measurement function and R is the measurement noise. This function could be different for each locale or each platform but is assumed the same here to clarify notation.

The aim of the planning problem is to find the best states, X. To do this it would appear that we are forced to also jointly optimise for the assignments, K. However, PMHT treats these assignments as nuisance parameters (missing data) and uses EM to iteratively marginalise them out of the problem.

In the PMHT context, EM consists of two alternating steps that are iteratively repeated until convergence. In the E-step, the probability of each possible assignment of platform and time to a locale is determined using the previous platform state estimate. In the M-step, an auxiliary function is optimised to find the new platform states. The auxiliary function is the expectation of the joint log-likelihood over the missing data, which now takes the form:

$$\begin{aligned} Q(X, \Pi^\tau, \Pi^k | \hat{X}(i), \hat{\Pi}^\tau(i), \hat{\Pi}^k(i)) \\ = \sum_K \sum_\tau \Bigg\{ P(\tau, K | \hat{X}(i), Z; \hat{\Pi}^\tau(i), \hat{\Pi}^k(i)) \\ \times \log P(X, \tau, K, Z; \Pi^\tau, \Pi^k) \Bigg\}, \end{aligned} \tag{2}$$

where the summation is over all permutations of the assignment variables τ and K, and $\hat{X}(i)$ denotes the estimated state at the ith EM iteration.

Due to the independence assumptions

$$P(X, \tau, K, Z; \Pi^\tau, \Pi^k) = P(X)P(\tau; \Pi^\tau)P(K; \Pi^k)P(Z|X, \tau, K), \quad (3)$$

where

$$P(X) = \prod_{m=1}^{M} \psi_t^0(x_0^m) \prod_{t=1}^{T} \psi_t^m(x_t^m | x_{t-1}^m), \quad (4)$$

$$P(\tau; \Pi^\tau) = \prod_n \pi_{n\tau_n}^\tau, \quad (5)$$

$$P(K; \Pi^k) = \prod_n \pi_{nk_n}^k, \quad (6)$$

$$P(Z|X, \tau, K) = \prod_{n=1}^{N} \zeta(z_n | x_t^m)|_{t=\tau_n, m=k_n}. \quad (7)$$

The conditional probability of the missing data, $P(\tau, K|\hat{X}(i), Z; \hat{\Pi}^\tau(i), \hat{\Pi}^k(i))$, is represented by a set of weights, which are given by [6] as:

$$\begin{aligned} &P(\tau, K|\hat{X}, Z; \hat{\Pi}^\tau, \hat{\Pi}^k) \\ &= \frac{P\left(\hat{X}, \tau, K, Z; \hat{\Pi}^\tau, \hat{\Pi}^k\right)}{\sum_{\tau, K} P\left(\hat{X}, \tau, K, Z; \hat{\Pi}^\tau, \hat{\Pi}^k\right)} \\ &= \prod_{n=1}^{N} \frac{\hat{\pi}_{nt}^\tau \hat{\pi}_{nm}^k \zeta(z_n | \hat{x}_t^m)\Big|_{t=\tau_n, m=k_n}}{\sum_{r=1}^{T} \sum_{s=1}^{M} \hat{\pi}_{nr}^\tau \hat{\pi}_{ns}^k \zeta(z_n | \hat{x}_r^s)} \equiv \prod_{n=1}^{N} w_{n,\tau_n,k_n} \quad (8) \end{aligned}$$

where the iteration index (i) is suppressed for clarity.

Thus the conditional probability of the assignments is given by the product of individual per locale *weights*. Each weight, w_{n,τ_n,k_n}, is the normalised likelihood of the nth locale from platform k_n at time τ_n. The numerator of the weight is the product of the assignment priors and the positional locale likelihood.

Combining the two equations (3) and (8) leads to the auxiliary function to be maximised:

$$\begin{aligned} &Q(X, \Pi^\tau, \Pi^k | X(i), \Pi^\tau(i), \Pi^k(i)) \\ &= \log P(X) + \sum_{n,t,m} w_{ntm} \log \pi_{nt}^\tau + \sum_{n,t,m} w_{ntm} \log \pi_{nm}^k \\ &\quad + \sum_{n,t,m} w_{ntm} \log \zeta(z_n | x_t^m) \\ &\equiv Q_X + Q_\Pi^\tau + Q_\Pi^k \quad (9) \end{aligned}$$

The term Q_Π^τ in (9) is given by

$$Q_\Pi^\tau \equiv \sum_{n,t,m} w_{ntm} \log \pi_{nt}^\tau,$$

and is similar to that of the standard multi-sensor PMHT [12]. It is maximised subject to the constraint that $\sum_t \pi_{nt}^\tau = 1$ using a Lagrangian, resulting in the updated prior estimate

$$\pi_{nt}^\tau(i+1) = \sum_{m=1}^{M} w_{ntm}, \quad (10)$$

i.e. the weights' relative frequency for time t.

Similarly, the Q_Π^k term results in a relative frequency estimate for the platform to locale assignment prior

$$\pi_{nm}^k(i+1) = \sum_{t=1}^{T} w_{ntm}. \quad (11)$$

The remaining term, Q_X, couples the platform states and the locales and is given by

$$Q_X \equiv \log P(X) + \sum_{n,t,m} w_{ntm} \log \zeta(z_n | x_t^m). \quad (12)$$

For a Gaussian penalty function, $\zeta(z_n | x_t^m)$, it can be shown that this function is equivalent to the log likelihood of a tracking problem with known data association [13],

$$Q_X \equiv \log P(X) + \sum_{t,m} \log \tilde{\zeta}(\tilde{z}_t^m | x_t^m), \quad (13)$$

where the synthetic locale, $\tilde{z}_t^m$, is given by

$$\tilde{z}_t^m = \frac{1}{\sum_{n=1}^{N} w_{ntm}} \sum_{n=1}^{N} w_{ntm} z_n, \quad (14)$$

and the synthetic locale function, $\tilde{\zeta}(\cdot)$, is a Gaussian distributed random variable with the same mean as the true locale function and a variance that is a scaled version of the sensor locale variance, R,

$$\tilde{R}_t^m = \frac{1}{\sum_{n=1}^{N} w_{ntm}} R. \quad (15)$$

The path for platform m is now refined by smoothing the synthetic locales and covariances. The planning algorithm consists of iteratively calculating assignment weights, w_{ntm}, and estimating the platform paths and assignment priors until convergence.

III. Non-Homogeneous Locales

The previous section described the path planning algorithm with a set of locales that were all equally important. In the examples provided in [6], [9] these locales were generally placed as a uniform grid. However, there are many applications where not all locations in the search space will be equally important. We now demonstrate a non-homogeneous locale map based on a model of locale *priority*.

Suppose a particular locale, s was more important than the others. This means that we would like the output of the path planning algorithm to place higher emphasis on passing close to the location s than other locations in the search space. There are two obvious ways in which the path planner could be influenced to achieve this. The first is to place duplicate locales at s. If there is more than one locale then this will have the

same effect as collecting two sensor observations at a certain location: the resulting state estimate will be closer to s than if there was only one. Alternatively, the fitting covariance R could be scaled for locale s, analogous to the assertion that this locale is more accurate than the others. We will follow the first method.

Suppose that a certain locale s is duplicated, so that there are now $N+1$ locales, $[1 \ldots N] \cup s$. This does not change the association weights, since they are independent for each locale. What changes is the range of the summand in the locale contribution to Q_X in (12). The effect of this is to duplicate the term due to locale s in the synthetic locale and covariance, namely (14) becomes

$$\begin{aligned} \tilde{z}_t^m &= \frac{\sum_{n=1}^{N+1} w_{ntm} z_n}{\sum_{n=1}^{N+1} w_{ntm}} \\ &= \frac{\left(\sum_{n=1}^{N} w_{ntm} z_n\right) + w_{stm} z_s}{\left(\sum_{n=1}^{N} w_{ntm}\right) + w_{stm}}, \end{aligned}$$

Similarly for the synthetic covariance in (15) becomes

$$\tilde{R}_t^m = \frac{1}{\left(\sum_{n=1}^{N} w_{ntm}\right) + w_{stm}} R.$$

The state estimation proceeds as previously and the resulting path will be influenced to pass closer to z_s.

The intuitive extension of this is now to place an integer number of copies of each locale, η_n, with a higher value of η_n where the locales have higher priority. As before, each of these duplicates has the same weight as z_n, so the synthetic locale (14) and covariance (15) become

$$\tilde{z}_t^m = \frac{\sum_{n=1}^{N} w_{ntm} \eta_n z_n}{\sum_{n=1}^{N} w_{ntm} \eta_n}, \tag{16}$$

$$\tilde{R}_t^m = \frac{1}{\sum_{n=1}^{N} w_{ntm} \eta_n} R. \tag{17}$$

Note that by introducing the priority η_n the effect is to reduce the synthetic covariance $\tilde{R}_t^m$ since $\sum_{n=1}^{N} w_{ntm}\eta_n > \sum_{n=1}^{N} w_{ntm}$ this can be compensated for by using a larger initial value of R, although this will influence the weights.

IV. PMHT-pp with Priority as a Continuous Density

Although the variable η_n represents a count of the number of duplicates of z_n the resulting synthetic locale and covariance in (16) and (17) could be evaluated for non-integer values of η_n. Accordingly, we will now refer to η_n as the *priority* of locale n and allow it to take on a positive semi-definite continuous value. This could be achieved rigorously by defining $\bar{\eta}_n = \lfloor \eta_n/\hbar^2 \rfloor$ and taking the limit as $\hbar^2 \to 0$ in the same manner as the derivation of Histogram-PMHT [14]. However, this proof is omitted due to space limitations.

Suppose now that the exploration area, $\mathcal{A}$, is a region in the plane and there is a known continuous priority intensity, $\rho(z) \geq 0 \forall z \in \mathcal{A} \subset \Re^2$.

Define a uniform grid of N pixels in the exploration area with pixel size Δ^2. Let the center of pixel n be z_n and the priority of pixel n be $\eta_n = \Delta^2 \rho(z_n)$. Clearly η_n is a piecewise constant approximation to $\rho(z)$. If $\rho(z_n)$ is the intensity of a Poisson point process, then η_n is the expected number of locales in the area of pixel z_n. Further we could interpret $\{\eta_n, z_n\}$ as a sample approximation of the intensity

$$\rho(z) \approx \sum_{n=1}^{N} \delta(z - z_n)\eta_n, \tag{18}$$

where $\delta(z)$ is the Dirac delta function.

Redefine the weight notation from w_{ntm} to $w_{tm}(z_n)$, which gives us the ability to represent weights for any $z \in \mathcal{A}$, i.e.

$$w_{tm}(z) = \frac{\hat{\pi}_{nm}^k \hat{\pi}_{nt}^\tau \zeta(z|\hat{x}_t^m)}{\sum_{r=1}^{T}\sum_{s=1}^{M} \hat{\pi}_{nr}^\tau \hat{\pi}_{ns}^k \zeta(z|\hat{x}_r^s)}. \tag{19}$$

The advantage in using this continuous valued η_n is that priorities can be balanced between locales without leading to a deflation of $\tilde{R}_t^m$.

Consider now the state dependent term from the auxiliary function, i.e. Q_X given by the modification of (12) to include the priority

$$\begin{aligned} Q_X &= \log P(X) + \sum_{n=1}^{N}\sum_{t=1}^{T}\sum_{m=1}^{M} \eta_n w_{tm}(z_n) \log \zeta(z_n|x_t^m), \\ &= \log P(X) + \sum_{t=1}^{T}\sum_{m=1}^{M} q_{tm}. \end{aligned} \tag{20}$$

where q_{tm} represents the locale contribution to the auxiliary function for platform m at time t. Recalling that ζ is a multivariate Gaussian with mean Hx_t^m and covariance R, so

$$\log \zeta(z_n|x_t^m) = \mathsf{const} - \frac{1}{2}(z_n - Hx_t^m)^\mathsf{T} R^{-1}(z_n - Hx_t^m), \tag{21}$$

where the constant term is due to the normalising term in the Gaussian pdf and is common for all locales and platforms.

We now substitute (21) into (20) and take the limit of the auxiliary function as $\Delta^2 \to 0$. It suffices to consider each q_{tm} individually as shown in equations (22) through (25) below. The constant term in these equations changes from one step to the next but contains only irrelevant terms that are independent from the target state.

The progression from summation to integral in (23) follows the Riemann notion of integration, which is sufficient for the types of functions we expect for $\rho(z)$ and $w_{tm}(z)$. The new synthetic locale and covariance implicitly defined by (24) and (25) are given by

$$\tilde{z}_t^m = \frac{\int w_{tm}(z)\ \rho(z)\ z\ dz}{\int w_{tm}(z)\ \rho(z)\ dz} \tag{26}$$

$$\lim_{\Delta^2\to 0} q_{tm} = \lim_{\Delta^2\to 0}\sum_{n=1}^{N}\eta_n w_{tm}(z_n)\Big\{\mathsf{const} - \frac{1}{2}(z_n - Hx_t^m)^\mathsf{T} R^{-1}(z_n - Hx_t^m)\Big\} \tag{22}$$

$$= \mathsf{const} - \frac{1}{2}\lim_{\Delta^2\to 0}\sum_{n=1}^{N}\Delta^2\rho(z_n)w_{tm}(z_n)\Big\{(z_n)^\mathsf{T} R^{-1}z_n - 2(Hx_t^m)^\mathsf{T} R^{-1}z_n + (Hx_t^m)^\mathsf{T} R^{-1}(Hx_t^m)\Big\}$$

$$= \mathsf{const} + (Hx_t^m)^\mathsf{T} R^{-1}\Big\{\lim_{\Delta^2\to 0}\sum_{n=1}^{N}\Delta^2\rho(z_n)w_{tm}(z_n)z_n\Big\} - \frac{1}{2}\Big\{\lim_{\Delta^2\to 0}\sum_{n=1}^{N}\Delta^2\rho(z_n)w_{tm}(z_n)\Big\}(Hx_t^m)^\mathsf{T} R^{-1}(Hx_t^m)$$

$$= \mathsf{const} + (Hx_t^m)^\mathsf{T} R^{-1}\Big\{\int_{\mathcal{A}}\rho(z)\ w_{tm}(z)\ z\ dz\Big\} - \frac{1}{2}\Big\{\int_{\mathcal{A}}\rho(z)\ w_{tm}(z)\ dz\Big\}(Hx_t^m)^\mathsf{T} R^{-1}(Hx_t^m) \tag{23}$$

$$= \mathsf{const} + (Hx_t^m)^\mathsf{T}\left(\tilde{R}_t^m\right)^{-1}\tilde{z}_t^m - \frac{1}{2}(Hx_t^m)^\mathsf{T}\left(\tilde{R}_t^m\right)^{-1}(Hx_t^m) \tag{24}$$

$$= \mathsf{const} - \frac{1}{2}(\tilde{z}_t^m - Hx_t^m)^\mathsf{T}\left(\tilde{R}_t^m\right)^{-1}(\tilde{z}_t^m - Hx_t^m) \tag{25}$$

$$\tilde{R}_t^m = \frac{1}{\displaystyle\int w_{tm}(z)\ \rho(z)\ dz}R. \tag{27}$$

Although we have arrived at this point through the limit of a uniform grid of locales, the same result could be obtained for a less regular configuration in the same way that Riemann integration need not be derived from a set of intervals with equal support. Using something other than the uniform grid would result in a locale-dependent Δ_n^2, but the limit as all of these approach zero would be the same.

The resulting algorithm no longer relies on a set of artificial points in the exploration region to guide the trajectories. Moreover it even provides a satisfying interpretation of the original method: the discrete grid of locales used to define the exploration region in [6] can be seen to be a numerical approximation to the path planning solution for a uniform priority intensity map. We note that the integrals in (26) and (27) do not appear to be tractable since $w_{tm}(z)$ is the ratio of a Gaussian to a sum of Gaussians. So it is likely that a numerical integral approximation would need to be used resulting in essentially the algorithm in section III.

The result described above assumes a constant priority intensity function, $\rho(z)$. An important extension would be to make $\rho(z)$ time-varying and dependent on the target state. This then would allow the intensity function to actually represent the expected number of targets in a region, conditioned on the sensor data, i.e. the PHD. In this case, the path planning solution could solve a much more general sensor resource scheduling problem.

V. Simulation Example

The difference between the existing PMHT-pp method presented in [6] and the new method derived in this paper is now illustrated through a simple simulation. We refer to the new method as non-homogeneous PMHT-pp.

Assume that the exploration area $\mathcal{A}$ is a 10 unit square box. The PMHT-pp method guides a single platform through the area by constructing a 10×10 uniform grid of discrete locales. Whereas the non-homogeneous PMHT-pp is guided by an intensity function that is a sum of two Gaussian components on a uniform pedestal,

$$\rho(z) = 0.01 + \sum_{j=1}^{2}\exp\Big\{-\frac{1}{2}(z-\mu_j)^\mathsf{T}\Sigma_j^{-1}(z-\mu_j)\Big\}, \tag{28}$$

with $\mu_1 = [8,3]^\mathsf{T}$, $\Sigma_1 = 0.25I$, $\mu_2 = [4,8]^\mathsf{T}$ and $\Sigma_2 = \mathsf{diag}(16, 0.8)$. The priority intensity contains an isolated point of high priority represented by the first component and a narrow ridge of high priority represented by the second.

The platform state was assumed to follow a constant acceleration model independently in X and Y. Thus the state evolution process is defined by

$$\psi(x_t|x_{t-1}) = \mathcal{N}\left(x_t; Fx_t, Q\right), \tag{29}$$

where $\mathcal{N}(t;\mu,\Sigma)$ is a multivariate Gaussian,

$$F = \begin{bmatrix} F_3 & 0 \\ 0 & F_3 \end{bmatrix} \quad \text{with} \quad F_3 = \begin{bmatrix} 1 & 1 & 0.5 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix},$$

and $Q = 10^{-8}I$. The fitting penalty (analogous measurement probability density) function was linear and Gaussian,

$$\zeta(z_n|x_t) = \mathcal{N}\left(z_n; Hx_t, R\right), \tag{30}$$

with $H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$ and $R = 10^{-2}I$.

$T = 500$ time points were used. The platform was initialised at position $[0, 0]$ with zero speed and zero acceleration.

Figure 1 shows the output of the discrete-locale PMHT-pp over the uniform grid. A trajectory is created that spans the grid of locales. Since the algorithm is not aware of the higher priority regions, the path pays no particular attention to them. In particular, the platform does not closely visit the area around $[8, 3]$. Figure 2 shows the output of the non-homogeneous PMHT-pp overlaid on the priority intensity map, as represented by (28). Clearly the paths in figures 1 and 2 are very different. The non-homogeneous PMHT-pp focuses on the locations of high priority and quickly skirts over the remaining area. Because the variance of the first intensity

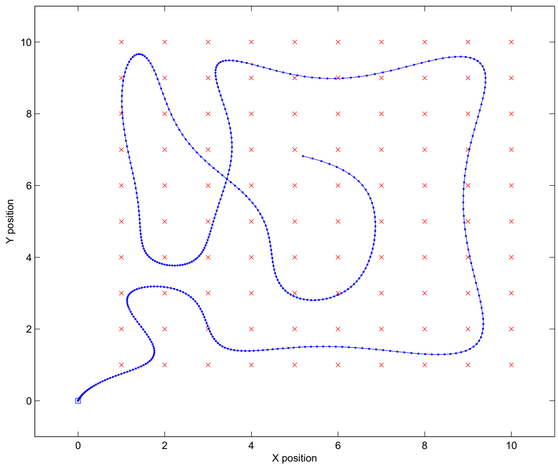

Fig. 1. Discrete-locale PMHT-pp trajectory

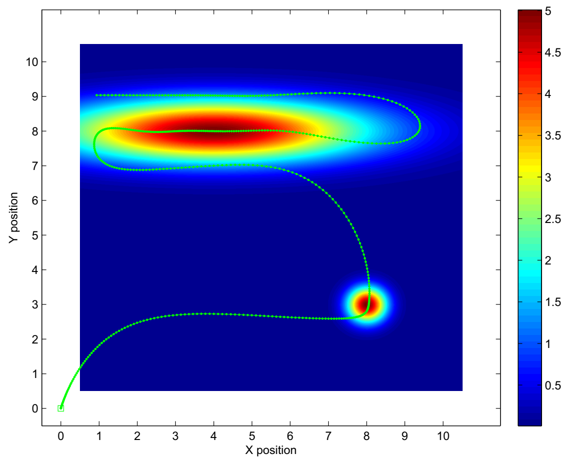

Fig. 2. Non-Homogeneous PMHT-pp trajectory with priority intensity map

component is so small, the platform trajectory passes directly through its centre. The path then makes several passes over the extended component.

VI. Conclusion

Previous work extended the PMHT data association method to handle the case where measurement time-stamps were unreliable or even completely unknown. This was then applied to the problem of coordinating multiple platforms through an exploration area in an algorithm referred to as PMHT-pp. This method relied on the introduction of discrete locales which were then associated with the platforms at specific times. PMHT-pp treats each of these discrete locales as equally important. In existing examples they have been either randomly placed with a uniform density, or distributed over a grid.

This paper has extended the PMHT-pp method to instead guide the platforms using a non-homogeneous intensity map. This allows for areas of higher importance and removes subjective parameters, such as the spacing of a discrete locale grid. In addition, the intensity guided PMHT-pp produces an intuitive result that the discrete-locale method is simply a numerical approximation to the intensity PMHT-pp.

Interestingly, the PMHT was developed for data association in target tracking, which can be considered to be a point-to-point assignment problem. In contrast, by removing the timing information and taking the limiting case of a point-process measurement model, the path planning problem considered in this paper effectively uses PMHT to assign a curve to a surface.

Although the algorithm was demonstrated in a path planning context, it is intuitive to see that the method has application in a wide range of problems, Fundamentally, the PMHT-pp algorithm efficiently associates a finite resource between multiple consumers over a long time horizon with dynamic constraints. It could be applied to many resource management problems. The time evolving intensity map could be used as a means to provide a sensor scheduling algorithm by using a Probability Hypothesis Density map of probable target locations.

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